Lec 20: 04/06/2010

Recombination Epoch:

As we have seen, neutrinos (and WIMPs) decouple at $t \sim 1$ sec ($T \sim 1$ MeV). Henceforth, we have a coupled plasma of baryons (mainly ionized H and ^4He), electrons and photons.

Photons scatter off the free electrons, and hence the plasma is opaque to photons. As time goes on, the kinetic energy of electrons and protons decreases, as well as the temperature of photons. Eventually neutral Hydrogen atoms can form efficiently. After that, photons freely move without scattering of the free electrons. These are the CMB photons that we observe today.

We start with a quick account of recombination and the important processes involved.

Recombination (Preview):

There are two important processes in the coupled plasma of baryons, electrons and photons:

1) Electron photon scattering:

$e^- + \gamma \rightarrow e^- + \gamma$

At temperatures well below MeV the scatterings can be considered as Thomson scattering. The scattering rate is:

$\Gamma_\gamma = n_e \sigma_T$

Where n_e is the number density of free electrons, and σ_T is the cross section for Thomson scattering. σ_T can be calculated from classical electrodynamics, for which we find:

$\boxed{\sigma_T = \frac{2}{3} \times 10^{-24}\ cm^2}$ $\quad (\frac{2}{3}$ barn, 1 barn $\equiv 10^{-24}\ cm^2)$

(2) Electron proton combination and ionization of Hydrogen:

$p + e^- \leftrightarrow H + \gamma$

In thermal equilibrium we have:

$$n_p = 2\left(\frac{m_p T}{2\pi}\right)^{3/2} \exp\left(\frac{\mu_p - m_p}{T}\right) \quad (2\text{: degrees of freedom in } e^-)$$

$$n_e = 2\left(\frac{m_e T}{2\pi}\right)^{3/2} \exp\left(\frac{\mu_e - m_e}{T}\right) \quad (2\text{: " " " " } p)$$

$$n_H = 4\left(\frac{m_H T}{2\pi}\right)^{3/2} \exp\left(\frac{\mu_H - m_H}{T}\right) \quad (4\text{: " " " " } H)$$

$$n_\gamma = 2\,\frac{\zeta(3)}{\pi^2}\,T^3 \quad (2\text{: " " " " } \gamma)$$

Here μ_p, μ_e, μ_H are the chemical potential of p, e, H respectively.

A non-zero chemical potential is an indicator of a conserved charge that is carried by a particle. In the case of e^- the number is electric charge (and similarly for p). In the case of H the conserved number is the baryon number.

Note that photons have zero chemical potential as they do not carry any conserved number. To elucidate consider the following process:

$e^- + \gamma \rightarrow e^- + \gamma$

onservation of electric charge guarantees that the initial and final states have the same charg (+1 here). In the non-relativistic limit, no e^+e^- pairs can be produced, and hence one e^- in the initial state implies one e^- in the final state.

However, one can have processes with different number of photons in the initial and final states. For example:

$e^- + \gamma \to e^- + \gamma + \gamma$

That can happen as a higher order process in quantum electrodynamics.

Another point to note is the number of degrees of freedom in e^-, p, H, γ. e^- and p are spin $\frac{1}{2}$ fermions, and hence in the non-relativistic limit (when anti-particles cannot exist because of kinematics) each of them represents two degrees of freedom (spin up and spin down). H is a bound state of two spin-$\frac{1}{2}$ particles, hence has 4 degrees of freedom. Also, γ represents two degrees of freedom (two polarization states of the electromagnetic radiation).

Ignoring ^4He (note that we have one ^4He for 12 H), we have:

$n_B = n_p + n_H$

Where n_B is number density of baryons. Also, neutrality of the plasma requires that:

$n_p = n_e$

If the total electric charge in the universe is initially zero, it will remain so since all of the Standard Model interactions preserve electric charge.

We also have:

$\mu_e + \mu_p = \mu_H$

Physically, this means that one electron and one proton combine to form one H (and one H is ionized to one e^- and one p).

Using this expression, and those for the number density of e^-, p, H we find:

$$n_H = n_p n_e \left(\frac{m_e T}{2\pi}\right)^{-3/2} \exp\left(\frac{B}{T}\right) \qquad (*)$$

Where:

$$B \equiv m_p + m_e - m_H = 13.6\ eV$$

The fractional ionization X_e is defined as:

$$X_e = \frac{n_p}{n_B}$$

In thermal equilibrium it is given by:

$$\boxed{\frac{1 - X_e^{eq}}{(X_e^{eq})^2} = \frac{4\sqrt{2}\,\zeta(3)}{\sqrt{\pi}}\,\eta \left(\frac{T}{m_e}\right)^{3/2} \exp\left(\frac{B}{T}\right)}$$

Where $\eta = \frac{n_B}{n_\gamma} \sim 6\times 10^{-10}$. This is the "Saha ionization equation", which can be found from $(*)$ and the expression given earlier for n_p, n_e (as well as $n_p = n_e$ and $n_B = n_H + n_p$).

Note that as $T \to 0$ we have $X_e^{eq} \to 0$. Like any other process in an expanding universe, $p + e^- \leftrightarrow H + \gamma$ also drops out of equilibrium. Thus X_e does not follow its equilibrium value X_e^{eq} all the way to $T = 0$. The residual ionization fraction X_∞ (X_e at $t \to \infty$) is found to be:

$$\boxed{X_\infty \simeq 3 \times 10^{-3}}$$

If we define recombination time as that when $X_e^{eq} \sim 10\%$, then we can find T_{rec} by solving the Saha ionization equation:

$$T_{rec} \sim (1200\text{–}1400)\, T_0 \sim 0.3 \text{ eV}$$

Where T_0 is the universe temperature at the present time. This results in a redshift z_{rec}:

$$\boxed{1 + z_{rec} = \frac{T_{rec}}{T_0} \Rightarrow z_{rec} \sim 1200\text{–}1400}$$

As a consistency check, note that X_e at recombination (as defined above) is $\gg X_\infty$. Thus it had its equilibrium value indeed, which follows the Saha ionization equation.

Next, we will give a more detailed and careful treatment of recombination. This is quite an important epoch, and CMB provides a snapshot of the universe at that time.